Cake Filtration Simulation

Filtration models can be used to analyze and optimize filtration operations. A cake filtration equation, which describes the variation in filtration flow rate as a function of operating parameters and physical properties, can be derived by applying Darcy's law to the filter cake and medium [1]. The cake filtration equation can be written as follows,

\[
\frac{dV}{dt} = \frac{\Delta P}{\mu c \alpha V} + \frac{\mu R_m}{A}
\]

(1)

where \( t \) is filtration time, \( V \) is filtrate volume, \( \mu \) is liquid viscosity, \( \alpha \) is the specific resistance of the cake, \( c \) is the dry cake mass per unit volume of filtrate, \( A \) is the filtration area, \( R_m \) is the filter medium resistance, and \( \Delta P \) is the combined pressure drop over the cake and medium. Several different modes of operation are possible. These include constant rate filtration, which corresponds to operation with a positive displacement pump, and constant pressure filtration, which reflects the typical mode of operation for vacuum filtration. In the latter case, the filtration rate decreases with time as the filter cake builds up. For compressible cakes, the variation in cake properties with time must also be considered in developing a model of the filtration operation. The filtration simulation tool incorporates models and solution methods described in the literature [1-3]. The principal features of these models are presented below.

Constant Rate, Incompressible Cake Filtration

For constant rate filtration, the cake filtration equation can be rearranged to obtain a linear relationship between the filtration pressure and filtrate volume,

\[
\Delta P = \left( \frac{\mu c \alpha q}{A^2} \right) V + \left( \frac{\mu R_m q}{A} \right)
\]

(2)

where \( q = \frac{dV}{dt} \) is the constant filtrate flow rate. The specific cake resistance and medium resistance can thus be determined from the slope and intercept of an experimental plot of filtration pressure against filtrate volume.
Constant Pressure, Incompressible Cake Filtration

The performance of a filter being operated under constant pressure can be described by integrating and rearranging the cake filtration equation to obtain the linearized parabolic rate law equation,

\[
\frac{t}{V} = \frac{\mu_{ac} c}{2 A^2 \Delta P} V + \frac{\mu R_n}{A \Delta P}
\]

(3)

In this case, the specific cake resistance and medium resistance can be determined experimentally from a plot of filtrate time over volume against filtrate volume.

Constant Rate, Compressible Cake Filtration

When analyzing compressible filter cakes it is important to consider the variation in cake properties during the filtration process. This can be achieved by modeling the deposition of the cake, layer by layer, with the cake properties being recalculated at each step [2]. For a constant rate filtration, the pressure drop required to maintain a constant flow will increase as the filtration progresses. The increasing pressure will result in a compaction of the filter cake, with a consequent increase in the solids concentration and specific resistance. The variation in cake properties with pressure can be modeled using empirical relationships such as those given below,

\[
\alpha_{av} = \alpha_{o} (1 - n) \Delta P_{av}^n,
\]

(4)

\[
C_{av} = C_{o} (1 - u) \Delta P_{av}^u,
\]

(5)

where \(\alpha_o\) and \(C_o\) are average values, \(\Delta P\) is the pressure drop across the cake, and \(\alpha\), \(C\), \(n\) and \(u\) are empirical constants. The simulation tool models the conditions for the formation of a new layer of filter cake. The pressure drop across the newly formed layer can be calculated from Darcy’s law,

\[
\frac{\Delta P}{L_a} = \mu \alpha_{av} \rho C_{av} q a A
\]

(6)
where \( q \) is the filtrate flow rate and \( \Delta P \) is the pressure drop across the newly forming layer. The thickness \( L_n \) of the newly deposited cake can be determined from a mass balance,

\[
L_n A \rho_s C_a = Q C_s \delta t \rho, \tag{7}
\]

where \( Q \) is the feed rate of suspension and \( C_s \) is the feed concentration expressed as a volume fraction. This can be calculated from the mass fraction \( s \) using the conversion

\[
C_s = \left( 1 + \frac{(1 - s) \rho_s}{s \rho} \right)^{-1}.
\]

The time \( \delta t \) required for the deposition of the new cake layer can be calculated by combining the pressure drop and mass balance equations to obtain the following equation

\[
\delta t = \Delta P_s \left( A^2 / \mu \alpha_s \rho_s q Q C_a \right). \tag{8}
\]

The filtrate flow rate can be calculated from the feed rate, \( q = Q \left[ 1 - \frac{C_s}{C_{av}} \right] \). The simulation calculation proceeds by incrementing the total pressure drop across the cake, computing the required filtration time, and recalculating the cake properties at each increment.

**Constant Pressure, Compressible Cake Filtration**

If medium resistance is neglected, then the entire pressure drop applies to the cake. In this case a simple solution is possible, since the cake concentration and specific resistance will be constant for the duration of the filtration step. For cases where the medium resistance is significant, the fraction of the overall pressure drop that applies to the cake will increase with time, resulting in an increase in cake concentration and resistance.

An iterative calculation scheme for modeling compressible cake filtration is proposed by Holdich [3]. The variation in cake resistance and concentration as a function of pressure can be modeled with empirical relationships as described in the previous section. The filtrate volume can be calculated by rewriting the rate law as a quadratic equation,
Here, $\bar{c}$ is the average mass of dry cake deposited per unit filtrate volume, which can be calculated from

$$\bar{c} = \frac{1}{s \rho} \left( \frac{1 - C_{\infty}}{C_{\infty} \rho_s} \right).$$

The filtrate flow rate can be calculated from the standard cake filtration equation,

$$q = \frac{dV}{dt} = \frac{1}{\frac{\mu \alpha_v \bar{c}}{A^2 \Delta P} V + \frac{\mu R_m}{A \Delta P}}.$$

The fraction of the overall pressure drop that applies to the filter cake can be calculated by subtracting the pressure drop over the medium from the overall pressure drop.

$$\Delta P_c = \Delta P - \frac{\mu R_m}{A} q.$$

The iterative solution process begins with the assumption that the entire pressure drop applies to the filter cake. This leads to an initial estimate of cake properties which can then be used to recalculate the pressure drop that applies to the filter cake. The new calculation incorporates the pressure drop across the medium. This leads in turn to a revised estimate of cake properties, and the iteration continues till the calculation of the pressure drop converges.

Given the filtrate flow rate and cake pressure drop values, the height of the filter cake and the cake concentration profile can be calculated from the following equations

$$L = \frac{\Delta P_c \left( \frac{y}{L} \right)^{1-a-n}}{\mu \alpha_v C_{\infty} \rho_s (1 - u - n) q} A,$$

$$C_y = C_\infty \Delta P_c^\alpha \left( \frac{y}{L} \right)^{1-a-n}.$$
Rappture Interface

The Rappture interface includes an option menu to choose from models for compressible and incompressible cakes, and operation modes for constant rate and constant pressure filtration. The input form includes fields for the entry of slurry parameters such as the viscosity, filter parameters such as the surface area and medium resistance, and cake properties such as the solids concentration and cake resistance (Figure 1). Additional parameters are required to describe the variation in cake properties with pressure for compressible cakes. Operating conditions such as the slurry feed rate for constant rate filtrations or the overall pressure drop for constant pressure filtrations must also be provided. Output plots include the filtrate flow rate for constant pressure filtration and the overall pressure drop for constant rate filtration. The filtrate time over volume against volume for constant pressure filtration is a straight line illustrating the linearized parabolic rate law.

The filtration simulation tool can be used to illustrate some of the phenomena that occur in compressible filter cakes. Figure 2 shows the increase in the filtrate flow rate for an ostensibly constant rate filtration that occurs as the filter cake is compressed, forcing some of the retained liquid out of the filter cake [2]. The solid concentration versus cake height is plotted at various time values for constant pressure compressible cake filtration. The plot in Figure 3 shows three profiles at processing times of 1 hour, 2 hours, and 3 hours respectively. As the processing time increases the fraction of the overall pressure drop that acts over the filter cake increases, resulting in an increase in the solid concentration at the surface of the filter medium [3].
NOMENCLATURE

\( A \) filtration area

\( C_s \) coefficient in cake concentration correlation

\( C_s \) volume fraction of solid in feed

\( C_y \) volume concentration at height \( y \)

\( c \) dry cake mass per unit volume of filtrate

\( \bar{c} \) average mass of dry cake deposited per unit filtrate volume

\( L \) height of filter cake

\( L_n \) thickness of newly deposited cake

\( n \) exponent in cake resistance correlation

\( \Delta P \) combined pressure drop over the cake and medium

\( \Delta P_c \) pressure drop across cake

\( \Delta P_n \) pressure drop across newly forming layer

\( Q \) feed rate of suspension

\( q \) filtrate flow rate

\( R_m \) filter medium resistance

\( s \) mass fraction of solid in feed

\( t \) filtration time

\( \delta t \) time required for deposition of new cake layer

\( u \) exponent in cake concentration correlation

\( V \) filtrate volume

\( \alpha \) specific resistance of cake

\( \alpha_o \) coefficient in cake resistance correlation

\( \mu \) liquid viscosity

\( \rho \) liquid density

\( \rho_s \) solid density
REFERENCES


2. R. Holdich, Software for simulation and modeling of separation processes, Department of Chemical Engineering, Loughborough University.

Figure 1 Simulation of cake filtration.
Figure 2 Increase in filtrate rate for compressible cake filtration.
Figure 3 Variation in solid concentration with cake height in constant pressure compressible cake filtration.