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Measurement of Powder Properties for Hopper Design[1]

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It is common practice in powder technology to run shear tests for characterising the flowability of powders. To measure these shear properties the flow factor tester of Jenike is mainly used. In this apparatus the state of stress cannot be determined completely and assumptions regarding the position of slip planes are necessary to evaluate the test results. Therefore this apparatus is less applicable in research work to get more information on the stress-strain behaviour of powders. In order to make such work possible a new apparatus, based on the ideas of Roscoe, has been developed, in which the states of stress and strain can be determined completely. Test results from measurements with both apparatuses will be compared and it will be shown, how the different results influence the design of hoppers.

Introduction

SHEAR tests are a common method in powder technology to characterise the flowability of granular materials and powders. A measure for the flowability is the flow function $ff_c = \sigma_1/f_c$ according to Jenike [1, 2, 3],[2] where σ_1 and f_c are the major principal stresses of the Mohr circles for stress shown in Fig. 1. τ (ordinate) is the shear stress necessary to shear a sample which is stressed by a normal compressive stress σ (abscissa). The envelope to Mohr circles as the representation of the states of stress in samples of the same initial bulk density ρ_b, which lead to flow, is called yield locus [1]. The circle with the major principal stress σ_1 (consolidating stress) touches the yield locus in its endpoint thus representing the state of steady flow where flow takes place without changes in stress and volume, whereas the circle with the principal stress f_c (unconfined yield strength) represents a limiting state of stress in a free surface, e.g. in an arch of material ahead of the discharge orifice of a hopper.

The envelope to all Mohr circles for stress leading to states of steady flow of samples of the same material but with different bulk densities, is nearly a straight line through the origin, called effective yield locus [1]. The angle δ appearing as 2δ in the Mohr circle representation (Fig. 1) is the angle between a slip plane in the bulk of material and a major principal plane. The intersection points of a yield locus with the coordinates give values of cohesion c and tensile strength t, both quantities dependent on the bulk density.

Shear Tests

Before describing the shear behavior in common shear apparatuses one should consider which shear tests are possible. Fig. 2 shows the two extreme cases. "a" is the case of pure Coulomb-friction without volume change. The other is a shear process (case b), where the shearing takes place homogeniously throughout the sample. Volume changes occurring in most cases of the flow of granular materials are possible. Most of the available shear apparatuses behave neither as in case a nor as in case b. The shear process in the flow factor tester of Jenike [1, 2, 4] is shown for comparison in Fig. 3 (c). Inside the lense-shaped shearing zone case b is nearly realised; outside this zone the material is moving as in case a.

At the Institute of Mechanical Process Engineering at the University of Karlsruhe we have run tests with the Flow Factor Tester of Jenike and another shear apparatus called simple shear apparatus, in which the shear process of Fig. 2 (b) is realised. Before reporting on the results some comments have to be made on the interpretation of shear test results.

In shear tests of the 3 cases shown in Fig. 2 the horizontal direction is for design reasons a direction without strain, i.e., the strain rate is $\dot{\epsilon}_h = 0$. In tests without volume change as in the endpoint of a yield locus the vertical strain is also zero ($\dot{\epsilon}_v = 0$). Thus the center of the Mohr circle for strain rate with $\dot{\epsilon}$ as normal strain rate (compressive strain positive) and $\dot{\gamma}/2$ as angular strain rate is in the origin of the $\dot{\epsilon}$, $\dot{\gamma}/2$-graph (Fig. 3(a)), and the angle between the vertical and the direction of the major principal strain rate $\dot{\epsilon}_1$ is $\pi/2$ in the Mohr circle, i.e., $\pi/4$ in the shear test. Iso-

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[1] Communication from the Institute of Mechanical Process Engineering, University of Karlsruhe, Germany.

[2] Numbers in brackets designate References at end of paper.

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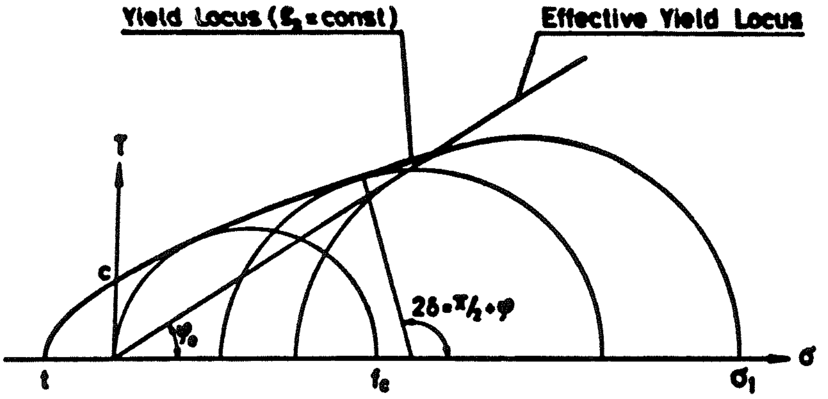


Fig. 1 Yield locus and effective yield locus

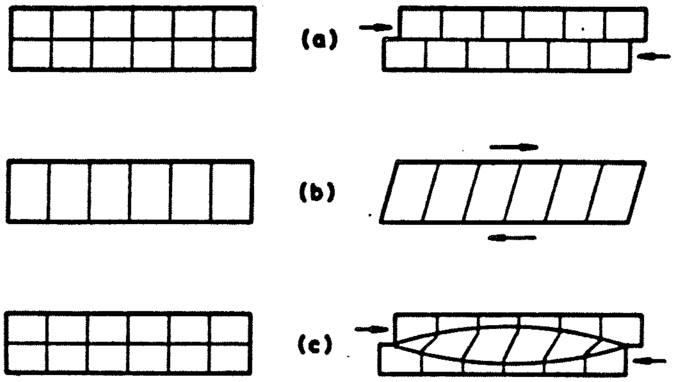


Fig. 2 Shear tests

tropy assumed, the directions of principal stress σ_1 and principal strain rate $\dot{\varepsilon}_1$ must coincide and σ_1 must act in a direction inclined at $\pi/4$ to the vertical. Therefore the stresses σ and τ acting in a horizontal plane are represented by point V in the Mohr circle for stress (Fig. 3 (*b*)) as the vertex of the circle. As can be seen point V is not identical with the tangential point T to the yield locus giving the direction of the slip planes in relation to the principal stress planes.

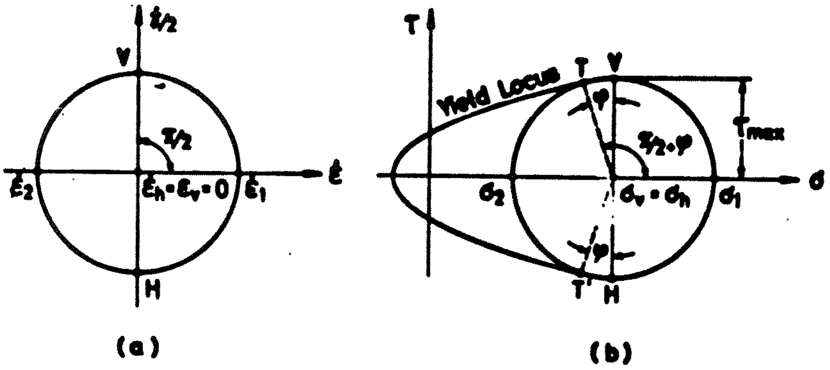
In tests yielding further points of a yield locus on the left of its end-point an increase in volume is observed: $\dot{\varepsilon}_1 + \dot{\varepsilon}_2 < 0$. Since there is no horizontal strain, the vertical strain rate must become $\dot{\varepsilon}_v < 0$ and the Mohr circle for strain rate of Fig. 4 a is obtained. In this case the angle between the vertical and the direction of $\dot{\varepsilon}_1$ is $1/2\,(\pi/2 + \nu)$. The value of ν can only be obtained, if the vertical strain rate $\dot{\varepsilon}_v$ and the angular strain rate $\dot{\gamma}/2$ are measured. The equivalent Mohr circle for stress is shown in Fig. 4 (*b*). As in Fig. 3 (*b*) points T and V are not identical.

If the material behaves isotropically, being one of the assumptions in the theory of Jenike [1], the horizontal shear plane, which in most apparatuses is forced upon the sample, is not identical with a slip plane. Knowing the angle φ (Figs. 3 (*b*) and 4 (*b*)) one can predict the directions of the slip planes in relation to each other and to the direction of principal stresses. In metals, where the yield locus is parallel to the σ-axis, points V and T coincide ($\varphi = \nu = 0$), and the two slip planes intersect an at angle of $\pi/2$. In the flow of bulk solids this angle is different from $\pi/2$ as can be seen, e.g., in the theory of Jenike [1] and from the experimental results of Cutress [5], who used X-rays. Thus $\varphi \neq \nu$ and points T and V cannot coincide.

Flow Factor Tester of Jenike

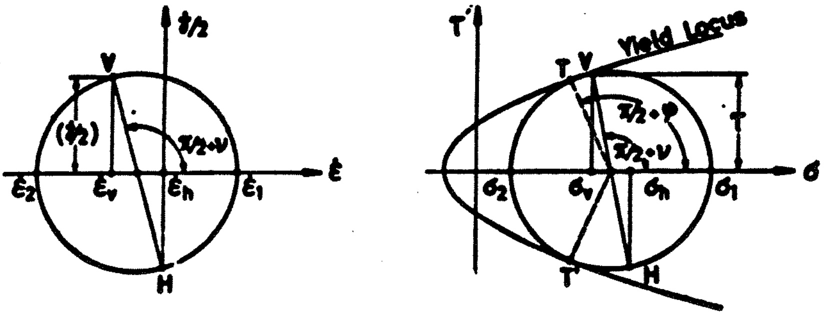
In the flow factor tester of Jenike shown schematically in Fig. 2(*c*) the normal compressive stress σ and the shear stress τ in the horizontal shear plane are measured, thus resulting in point V. Since an angular strain rate $\dot{\gamma}/2$ and a normal strain rate $\dot{\varepsilon}_v$ cannot be determined due to the unknown thickness and shape of the lense-shaped shearing zone a value for ν cannot be obtained and it is not possible to draw a complete Mohr circle without a further assumption. Jenike assumed that in shear tests yielding points on the left of the end-point of yield loci the horizontal plane is a slip plane ($T = V$ in Fig. 4(*b*)) and in tests yielding an end-point the horizontal is neither a slip plane ($T \neq V$ in Fig. 3(*b*)) nor a plane of maximum shear stress (point V in Fig. 3(*b*)), but lies between these two possibilities [4].

One aim of our investigations was to see how this disagreement in the assumptions for the behavior of granular materials in Jenikes tester influences the design of bins and hoppers. Another question was, in what way the preparation technique for the test specimen affects the results. In our tests with limestone with particle sizes < 15 μm (mean particle size of ~ 3μm) we found out that the preparation technique is without any influence as long as the



(a) (b)

Fig. 3 Mohr circles for strain rate (a) and stress (b) for shear tests without volume change



(a) (b)

Fig. 4 Mohr circles for strain rate (a) and stress (b) for shear tests with volume increase

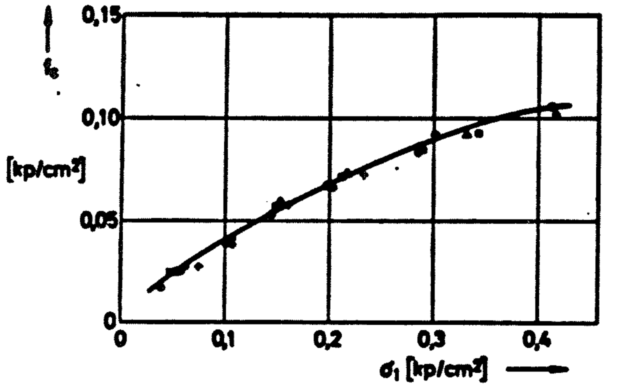


Fig. 5 Unconfined yield strength f_c versus consolidating stress σ_1 for tests with the flow factor tester

shear stress-shear strain-curves are interpreted correctly and the right conclusions are drawn from the graphs. The use of three-dimensional yield surfaces in a σ, τ, ϵ-space with ϵ as porosity is helpful. For more details see the original work [6].

In the first approach all test results are evaluated using the assumptions of Jenike, i.e., letting the horizontal plane be identical with slip planes with the exception of the tests leading to end-points of yield loci (see above). Each yield locus gives a value of the function $f_c(\sigma_1)$, the unconfined yield strength f_c in dependence of the consolidating stress σ_1. The results are plotted in a graph with f_c as ordinate and σ_1 as abscissa (Fig. 5). Four different preparation techniques ($a - d$) have been used and 3 different people have run the tests [6]:

(a) Exertion of twisting motions to the top of the sample under normal load V and preshearing under the same load $N = V$ (○, ◇, ●).

(b) No twisting, consolidation using normal load $V_1 > N$ and no preshearing (+).

(c) Twisting under normal load $V_2 < V_1$ and preshearing under normal load $N < V_2$(□).

(d) As c, but with more twisting motions and a smaller load V_3(△).

As can be seen from the graph, one single curve can be drawn through the points.

Simple Shear Apparatus

The flow factor tester of Jenike has mainly two disadvantages for research work:

(a) A horizontal shear direction is forced upon the sample, and it is assumed that the horizontal plane is identical with a slip plane in the material at maximum value of the shear force (see above). During the shear process the direction of the major principal stress has to be rotated against the vertical from $\delta = 0$ at $\tau = 0$ to $\delta = \pi/4 + \varphi/2$ (Fig. 1). Whether this occurs or not, cannot be proved.

(b) Since the thickness and shape of the zone, where the shearing takes place, is unknown strain measurements are impossible.

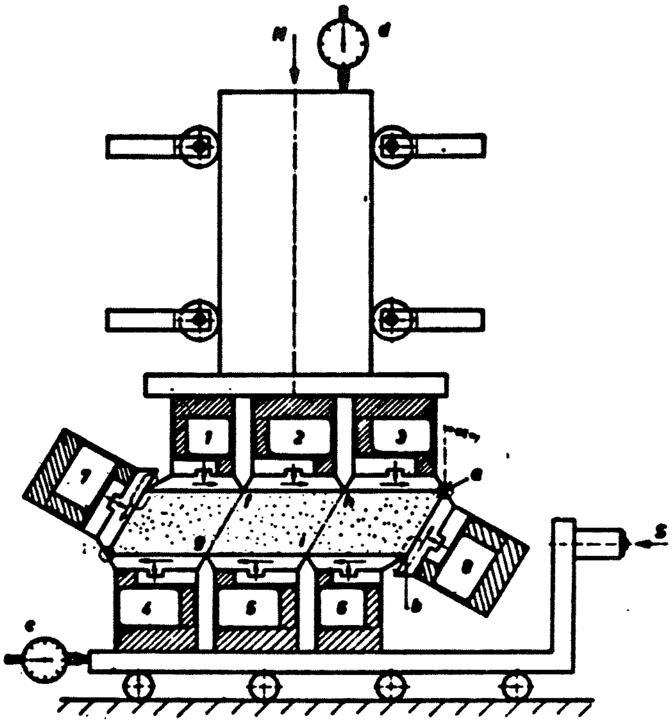


Fig. 6 Simple shear apparatus

A shear apparatus in which the states of stress and strain can be determined completely and independently is the simple shear apparatus of Roscoe [7, 8]. Since it was designed to meet the demands of soil mechanics some changes had to be made to make the apparatus effective for use in powder technology [6].

Fig. 6 shows a schematic drawing of the simple shear apparatus as existing in the Institute of Mechanical Process Engineering at the University of Karlsruhe. The sample of 10 by 10 cm^2 and about 2 cm high is stressed by a vertically guided stamp with a normal force N. On the bottom of the apparatus a shear force S is acting. This shear stress is uniformly transmitted layer by layer from the bottom to the top of the sample. Both side planes which are vertical before the application of the shear force S can be rotated about the points a and can move in the slits b. Thus the sample can be deformed in the same way as it would be without boundaries if the same shear stress is acting on each element

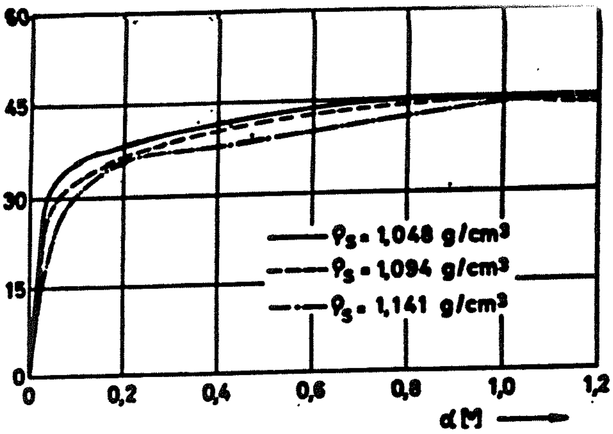


Fig. 7 Angle δ between the vertical and the direction of major principal stress versus shear angle α for tests without volume change

of the sample. A vertical strain and a change in the shear angle α can be read from the micrometers *c* and *d*. For measuring the stresses special load cells 1–8 with electric strain gages have been developed [6].

The state of stress in the inner third of the sample is needed for calculation purposes. To draw the Mohr circle for stress the stresses on the planes between f and g and h and i are determined from the stress readings of the load cells 1, 4, and 7 and 3, 6, and 8, respectively. Thus Mohr circles for stress and strain can be obtained completely and independently. It is outside the scope of this paper to describe the simple shear apparatus in detail, to explain the difficulties and necessary care in running tests and to report on the method of evaluating test results [6, 9].

Figs. 7 to 9 show some of the results which give an answer to the questions put forward in this paper. In Fig. 7 the angle δ between the vertical and the direction of major principal stress is plotted against the shear angle α. The three curves are results from tests on samples with different bulk densities ρ_s leading to end-points of yield loci without volume change. In all three cases a constant maximum angle of $\delta = \pi/4$ is reached, which is in accordance with the assumption of isotropy (see Fig. 3). In tests yielding further points of yield loci the angle δ is greater than $\pi/4$ but smaller than $\pi/4 + \varphi/2$ ($\pi/4 < \delta < \pi/4 + \varphi/2$), thus showing the behavior predicted in Fig. 4.

Drawing the greatest Mohr circles for stress of tests on samples with the same initial bulk density ρ_s in a τ, σ-graph, a yield locus is obtained as the envelope to these circles. Such a yield locus is shown in Fig. 8. The tensile strength *t* as the minor principal stress of the smallest Mohr circle is determined in special tensile strength measurements [6]. By plotting a circle through the origin tangential to the yield locus the unconfined yield strength f_c is obtained, which in connection with the major principal stress σ_1 of the Mohr circle through the end-point of the yield locus gives one point of function $f_c(\sigma_1)$.

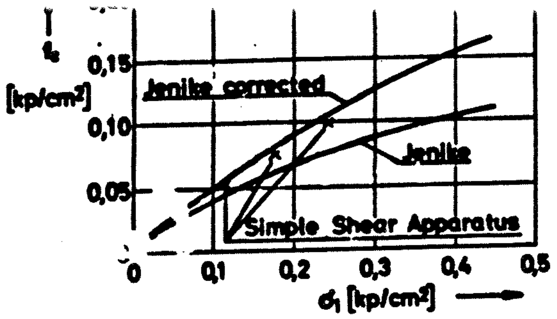


Fig. 9 Unconfined yield strength versus consolidating stress

In Fig. 9 the results of all tests are plotted in a f_c,σ_1-graph. The curve called Jenike is identical with that of Fig. 5. If the same test results with Jenikes tester are taken, but—contrary to the first approach—an isotropic behavior in the shear test is assumed, the measured values of τ and σ will give the vertex points of the Mohr circles for the tests leading to end-points of yield loci. By simplifying the procedure in assuming the same for all tests in a second approach (putting $\nu \rightarrow 0$ in Fig. 4(*b*)) all Mohr circles can be drawn in the same way and yield loci are obtained as envelopes to these circles. Since these yield loci have greater τ-values, f_c will also become greater and the curve called "Jenike corrected" must be located above the curve of Fig. 5.

Besides these two curves three values from tests with the simple shear apparatus are plotted in Fig. 9. These points lie above the Jenike-curve, as to be expected, but below the corrected Jenike-curve. This can be explained by the fact that in the second approach δ is assumed to be $\delta = \pi/4$ for all cases. However, the tests in the simple shear apparatus show that δ is $\pi/4 \leq \delta < (\pi/4 + \varphi/2)$, thus resulting in $\varphi > \nu \geq 0$ (see Fig 4). In accordance with the isotropic behavior the first approach underestimates the unconfined yield strength f_c and the second approach overestimates f_c.

Conclusions

The investigations have shown, that

1. Bulk solids, as they have been tested, show isotropic behavior up to the point where the state of flow is reached and
2. The tests in the flow factor tester give similar results to those in the simple shear apparatus, if interpreted correctly.

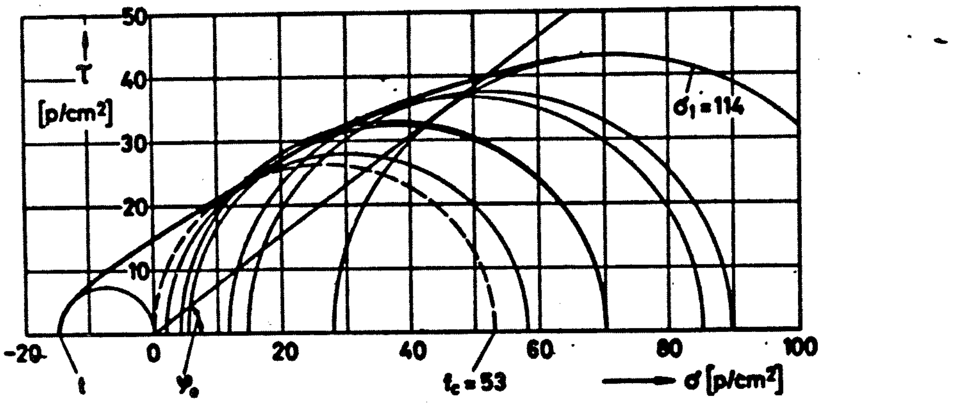


Fig. 8 Yield locus from tests with the simple shear apparatus (initial bulk density ρ_s = 1,048 g/cm³)

For the design of hoppers the following conclusions can be drawn:

1. The angles φ_e of the effective yield locus (Fig. 1) are in the tests with the simple shear apparatus insignificantly greater than those in the tests with Jenike's tester using Jenike's procedure for evaluation. Since the influence of the value of φ_e on the limiting values for mass flow is small, one can conclude that tests with the flow factor tester give correct results regarding the minimum inclination of hopper walls for mass flow to occur.
2. The unconfined yield strength f_c affects the minimum size of the discharge orifice making doming of material in the hopper impossible. The values of f_c and therefore the minimum sizes of the orifice become greater when using the correct results of the simple shear apparatus. In spite of lacking agreement the hoppers designed after Jenikes method are working without difficulties. The only conclusion can be that the safety factor entering the calculations is greater than the difference due to the other f_c-values of the simple shear apparatus. Since engineers are neither interested in overdesign and since the size of the safety factor is unknown, more research is planned in that direction.

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DISCUSSION

A. W. Jenike[3]

Dr. Schwedes modified the procedure which we recommend for the Jenike shear cell to have the results comply with those of Roscoe's simple shear cell. The author claims that this compliance ensures the correctness of results obtained on the Jenike tester. In so doing, the author assumes that Roscoe's shear cell is a perfect instrument, that, to quote, "shear stress is uniformly transmitted layer by layer from the bottom to the top of the sample." In fact, the late professor Roscoe never advanced such a claim. His numerous and painstaking experiments, backed by x-ray photography of the shearing material, invariably showed no homogeneity within the deforming material.

This discusser suggests that it is just as impossible to obtain homogeneous strain in a shear cell during expansion (shear of an overconsolidated sample) as it is to rupture a tensile steel specimen without necking. In the above tests, either a small local weakness within the sample or a slight concentration of applied stress leads to the initiation of failure within some narrow band of material. Along that band, material expands and, hence, grows weaker with respect to adjacent material thus reinforcing the nonhomogeneity of deformation and stress.

If Dr. Schwedes has been able to produce homogeneous strain in a modified version of Roscoe's shear cell, it would be very interesting to see the evidence of it.

Meantime, this discusser suggests that the users of the Jenike tester apply the procedure described in the instruction that accompanies the tester. This procedure has now been used in essentially unchanged form for some twelve years and the results have been confirmed in hundreds of industrial installations.

Author's Closure

Dr. Jenike writes in his contribution that homogeneous stress and homogeneous strain in a simple shear apparatus are very difficult to obtain. One should consider both cases separately, because a nonhomogeneous strain must not be connected with a nonhomogeneous stress.

The state of stress in the inner third of the sample, which is needed for calculation of the Mohr circle for stress, has been homogeneous for the following reasons [6]:

1 In most tests the normal and shear stresses on the bottom and top of the inner third of the sample had the same values;

2 Shear stresses on both side planes parallel to the plane of Fig. 6 were very small and could be neglected.

For force equilibrium reasons the state of stress is homogeneous and the shear stress is uniformly transmitted layer by layer from the bottom to the top of the sample. Roscoe did not say that homogeneous stress is impossible, he only stated that homogeneous strain in overconsolidated samples beyond the point of failure can not be obtained in shear tests.

During shear of critical consolidated samples the shear stress remained at a constant maximum value without a volume change. Investigations with γ-rays showed, that the samples have a homogeneous bulk density before shearing.[4] Since there was no volume change during the shear process there is no reason to be in doubt about the homogeneity of deformation. Overconsolidated samples underwent no appreciable volume change before the maximum value of shear stress was obtained, i.e., the homogeneous state of strain was kept up to the point, where the state of flow was reached. Beyond this point the deformation is no longer homogeneous and Dr. Jenike's remark about narrow bands of weaker material originating is correct.

To design hoppers the conditions of steady-state flow and of incipient failure must be known. Starting with homogeneous samples both states can be obtained in a simple shear apparatus without loosing the homogeneity of stress and strain. But one has to keep in mind that critical consolidated samples must be used to reach the critical state in all layers of the sample. Starting with an overconsolidated sample the critical state will only be obtained in a narrow band [7, 8].

To repeat the last paragraph of the author's paper it should be noted that in determining the minimum size of the discharge orifice after Jenike's method a safety factor enters the calculations the size of which is unknown. As long as this safety factor results in greater values for the discharge orifice than those obtained with the f_c-values of the simple shear apparatus the hoppers designed after Jenike's method ought to work without difficulties.

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